Absence of time-reversal symmetry breaking in association with the order parameter of Cooper pair in high T_c superconductivity

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Abstract

As an extension of our previous work on the holon pairing instability in the t-J Hamiltonian [Phys. Rev. B **66**, 054427 (2002)], we examine the orbital symmetries of holon pairing order parameters in high T_c superconductivity by examining the energy poles of t-matrix. We find that both s- and d-wave symmetries in holon pair order parameter occur at low lying energy states corresponding to the higher energy poles of t-matrix while only the s-wave symmetry appears at the lowest energy pole and that this results in the d-wave symmetry in the Cooper pair order which is a composite of the holon pair of s-wave symmetry and the spinon pair of d-wave symmetry below T_c. Finally we demonstrate that there exists no time-reversal symmetry breaking in association with the Cooper pair order parameter.

I. INTRODUCTION

At present the Cooper pair order parameter of $d_{x^2-y^2}$-wave symmetry for high T_c cuprates is generally accepted [1]. However, recent tunneling measurements on $YBa_2Cu_3O_{7-3}$ compound show that pairing symmetry changes from a pure $d_{x^2-y^2}$-wave symmetry to a mixture of d- and s- wave symmetries with the change of hole concentration [2–4] or with the variation of applied magnetic field [2]. Cooper pair order parameters of complex mixing such as $d_{x^2-y^2} + id_{xy}$ and $d_{x^2-y^2} + is$ break the time-reversal symmetry [2,4]. There have been some theoretical attempts to explain the observed thermal conductivity [5] in high T_c cuprates by resorting to the time-reversal symmetry breaking of the Cooper pair wave function [6,7]. On the other hand, the signatures for time-reversal symmetry breaking have also been found in the angle resolved photoemission spectroscopy [8], neutron scattering [9] and μ-SR measurements [10] below the pseudogap temperature T^*. Earlier, Varma [11] proposed that the fourfold pattern of circulating current inside the CuO_2 unit cell breaks the time-reversal symmetry. Chakravarty et al. [12] suggested that the time-reversal symmetry is broken by the d-density wave order which involves the circulating current around the Cu-O bond.

Recently we [14] reported the superconducting phase diagram by applying the Bethe-Salpeter equation to our earlier U(1) and SU(2) holon-pair boson theory [15] of t-J Hamiltonian and obtained an arch shape structure of superconducting transition temperature in agreement with experiments. The $d_{x^2-y^2}$-wave symmetry of Cooper pair can arise as a composite of the s-wave symmetry of the holon pair and the $d_{x^2-y^2}$-wave symmetry of spinon pair. In the previous t-matrix study we briefly reported a discussion on the d-wave symmetry of the Cooper pair order parameter at and below T_c. Here we present a detailed study on how the orbital symmetry of the holon pairing order parameters varies as a function of excitation energy below and above T_c. It is of great interest to examine whether the widely used t-J Hamiltonian can intrinsically predict the time reversal symmetry breaking by allowing a complex orbital mixing. The objectives of the present paper are two-fold. First we discuss the variation of the orbital symmetry for holon pair order parameter with excitation energy by examining the t-matrix pole. Second we examine whether there exists the time reversal symmetry breaking in association with the Cooper pair order parameter based on the U(1) and SU(2) slave-boson theories of the t-J Hamiltonian.

II. U(1) AND SU(2) SLAVE-BOSON REPRENSENTATIONS OF T-MATRIX BASED ON T-J HAMILTONIAN

We write the t-J Hamiltonian,

$$H = -t\sum(c^\dagger_{i\sigma}c_{j\sigma} + c.c.) + J\sum(\mathbf{S}_i\cdot\mathbf{S}_j - \frac{1}{4}n_in_j), \tag{1}$$

where $\mathbf{S}_i\cdot\mathbf{S}_j - \frac{1}{4}n_in_j = -\frac{1}{2}(c^\dagger_{i\downarrow}c^\dagger_{j\uparrow} - c^\dagger_{i\uparrow}c^\dagger_{j\downarrow})(c_{j\uparrow}c_{i\downarrow} - c_{j\downarrow}c_{i\uparrow})$. Here $\mathbf{S}_i$ is the electron spin operator at site i, $\mathbf{S}_i = \frac{1}{2}c^\dagger_{i\alpha}\sigma_{\alpha\beta}c_{i\beta}$ with $\sigma_{\alpha\beta}$, the Pauli spin matrix element and n_i, the electron number operator at site i, $n_i = c^\dagger_{i\sigma}c_{i\sigma}$. In the slave-boson representation [16–21] the electron annihilation operator c_σ of spin σ is written as a composite of spinon f_σ (spin 1/2 and charge 0 object) and holon b (spin 0 and $+e$ object) operators. That is, $c_\sigma = b^\dagger f_\sigma$ in the U(1) theory and $c_\alpha = \frac{1}{\sqrt{2}}h^\dagger\psi_\alpha$ in the SU(2) theory with $\alpha = 1,2$, where $f_\sigma(b)$ is the spinon(holon) annihilation operator in the U(1) theory, and $\psi_1 = \begin{pmatrix} f_1 \\ f_2^\dagger \end{pmatrix}$ and $\psi_2 = \begin{pmatrix} f_2 \\ -f_1^\dagger \end{pmatrix}$ and $h = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ are respectively the doublets of spinon and holon annihilation operators in the SU(2) theory. In the slave-boson representation, the Heisenberg interaction term is written $H_J = -\frac{J}{2}\sum b_ib_jb^\dagger_jb^\dagger_i(f^\dagger_{i\downarrow}f^\dagger_{j\uparrow} - f^\dagger_{i\uparrow}f^\dagger_{j\downarrow})(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})$ for the U(1) theory and $H_J = -\frac{J}{2}\sum(1+h^\dagger_ih_i)(1+h^\dagger_jh_j)(f^\dagger_{i2}f^\dagger_{j1} - f^\dagger_{i1}f^\dagger_{j2})(f_{j1}f_{i2} - f_{j2}f_{i1})$ for the SU(2) theory. In the above expressions the coupling between the charge (holon pair) and spin (spinon pair) degrees of freedom naturally arises from the composite nature of electron having both spin and charge.

After decomposition of the Heisenberg interaction term and proper Hubbard Stratonovich transformations, the effective mean field Hamiltonian of holon is derived to be [15], for the U(1) theory (see Appendix A for a derivation),

$$\begin{aligned} H^b_{t-J,U(1)} &= -t\sum\chi^*_{ij}b^\dagger_ib_j + c.c. \\ &- \frac{J}{2}\sum|\Delta^f_{ij}|^2b^\dagger_ib^\dagger_jb_jb_i - \mu\sum_i b^\dagger_ib_i, \end{aligned} \tag{2}$$

where $\chi_{ij} =< f^\dagger_{i\sigma}f_{j\sigma} + \frac{4t}{J_p}b^\dagger_ib_j >$ is the hopping order parameter and $\Delta^f_{ij} =< f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow} >$, the spinon pairing order parameter, and for the SU(2) slave-boson theory, (see Appendix B for a derivation),

$$H^b_{t-J,SU(2)} = -\frac{t}{2}\sum h_i^\dagger U_{ij} h_j + c.c.$$
$$-\frac{J}{2}\sum_{,\alpha,\beta} |\Delta^f_{ij}|^2 b^\dagger_{i\alpha} b^\dagger_{j\beta} b_{j\beta} b_{i\alpha} - \mu \sum_i h_i^\dagger h_i, \tag{3}$$

where $U_{i,j} = \begin{pmatrix} \chi^*_{ij} & -\Delta^f_{ij} \\ -\Delta^{f*}_{ij} & -\chi_{ij} \end{pmatrix}$ is the order parameter matrix of hopping order, χ_{ij} and spinon pairing order, Δ^f_{ij} with $\chi_{ij} =< f^\dagger_{i\sigma} f_{j\sigma} + \frac{2t}{J_p}(b^\dagger_{i1} b_{j1} - b^\dagger_{j2} b_{i2}) >$ and $\Delta^f_{ij} =< f_{j1} f_{i2} - f_{j2} f_{i1} >$.

From the Bethe-Salpeter equation [14], we obtain a matrix equation for the t-matrix, for the U(1) slave-boson theory (see Appendix C for a detailed derivation),

$$\sum_{\mathbf{k}''} \left(\delta_{\mathbf{k}',\mathbf{k}''} - m_{\mathbf{k}',\mathbf{k}''}(\mathbf{q}, q_0)\right) t_{\mathbf{k}'',\mathbf{k}}(\mathbf{q}, q_0) = v(\mathbf{k}' - \mathbf{k}), \tag{4}$$

where

$$m_{\mathbf{k}',\mathbf{k}''}(\mathbf{q}, q_0) = \frac{1}{N} v(\mathbf{k}' - \mathbf{k}'') \frac{n(\epsilon(\mathbf{k}'')) + e^{\beta\epsilon(-\mathbf{k}''+\mathbf{q})} n(\epsilon(-\mathbf{k}''+\mathbf{q}))}{iq_0 - (\epsilon(-\mathbf{k}''+\mathbf{q}) + \epsilon(\mathbf{k}''))}, \tag{5}$$

$v(\mathbf{k}) = -J|\Delta_f|^2\gamma_\mathbf{k}$, the momentum space representation of holon-holon interaction with $\gamma_\mathbf{k} = (\cos k_x + \cos k_y)$ and $n(\epsilon) = \frac{1}{e^{\beta\epsilon}-1}$, the boson distribution function. Similarly, we obtain, for the SU(2) slave-boson theory (see Appendix D for a derivation),

$$\sum_{\mathbf{k}'',\alpha'',\beta''} \left(\delta_{\mathbf{k}',\mathbf{k}''}\delta_{\alpha'\alpha''}\delta_{\beta'\beta''} - m^{\alpha'\beta'\alpha''\beta''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{q}, q_0)\right) t^{\alpha''\beta''\alpha\beta}_{\mathbf{k}'',\mathbf{k}}(\mathbf{q}, q_0)$$
$$= v(\mathbf{k}' - \mathbf{k})\delta_{\alpha'\alpha}\delta_{\beta'\beta}, \tag{6}$$

where

$$m^{\alpha'\beta'\alpha''\beta''}_{\mathbf{k}',\mathbf{k}''}(\mathbf{q}, q_0) \equiv \frac{1}{N}\sum_{\alpha'_1\beta'_1} v(\mathbf{k}' - \mathbf{k}'') \frac{n(E_{\alpha'_1}(\mathbf{k}'')) + e^{\beta E_{\beta'_1}(-\mathbf{k}''+\mathbf{q})} n(E_{\beta'_1}(-\mathbf{k}''+\mathbf{q}))}{iq_0 - (E_{\alpha'_1}(\mathbf{k}'') + E_{\beta'_1}(-\mathbf{k}''+\mathbf{q}))} \times$$
$$U_{\alpha'\alpha'_1}(\mathbf{k}'') U_{\beta'\beta'_1}(-\mathbf{k}''+\mathbf{q}) U^\dagger_{\alpha'_1\alpha''}(\mathbf{k}'') U^\dagger_{\beta'_1\beta''}(-\mathbf{k}''+\mathbf{q}). \tag{7}$$

Here $E_1(\mathbf{k}) = E_\mathbf{k} - \mu$ and $E_2(\mathbf{k}) = -E_\mathbf{k} - \mu$ are the energy dispersions of upper and lower bands of holons with $E_\mathbf{k} = t\sqrt{(\chi\gamma_\mathbf{k})^2 + (\Delta_f\varphi_\mathbf{k})^2}$ and $\varphi_\mathbf{k} = (\cos k_x - \cos k_y)$. $U_{\alpha\beta}(\mathbf{k}) = \begin{pmatrix} u_\mathbf{k} & -v_\mathbf{k} \\ v_\mathbf{k} & u_\mathbf{k} \end{pmatrix}$ is the unitary matrix which diagonalizes the one-body holon Hamiltonian with $u_\mathbf{k} = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{t\chi\gamma_\mathbf{k}}{E_\mathbf{k}}}$ and $v_\mathbf{k} = \frac{sgn(\varphi_k)}{\sqrt{2}}\sqrt{1 + \frac{t\chi\gamma_\mathbf{k}}{E_\mathbf{k}}}$. (Here $\mathrm{sgn}(\varphi_k)$ denotes the sign of $(\cos k_x - \cos k_y)$.) It is noted that the t-matrices, $t_{\mathbf{k}',\mathbf{k}}(\mathbf{q}, q_0)$ and $t^{\alpha'\beta'\alpha\beta}_{\mathbf{k}',\mathbf{k}}(\mathbf{q}, q_0)$ for the U(1) and SU(2)

theories respectively are independent of the Matsubara frequencies $k_0^{'}$ and k_0, owing to the consideration of instantaneous holon-holon interaction, $v(\mathbf{k}^{'} - \mathbf{k})$.

The t-matrices are numerically evaluated from the use of Eqs.(4) and (6) at each temperature and hole doping concentration. The hopping and spinon pairing order parameters in Eqs.(2) and (3) are the saddle point values evaluated from the usual partition functions involving the functional integrals of slave-boson representation. [15] Using the matrix equations (4) and (6), the poles of the t-matrices are searched for as a function of energy q_0 with $\mathbf{q} = 0$, i.e., the zero momentum of the holon pair by using

$$\sum_{\mathbf{k}^{'}} t_{\mathbf{k},\mathbf{k}^{'}}(q_0, \mathbf{q} = 0) W(q_0, \mathbf{k}^{'}) = \lambda W(q_0, \mathbf{k}), \tag{8}$$

for the U(1) theory, where $W(q_0, \mathbf{k})$ is the eigenvector and λ, the eigenvalue and

$$\sum_{\mathbf{k}^{'}, \alpha^{'}, \beta^{'}} t_{\mathbf{k},\mathbf{k}^{'}}^{\alpha\beta\alpha^{'}\beta^{'}}(q_0, \mathbf{q} = 0) W_{\alpha^{'}\beta^{'}}(q_0, \mathbf{k}^{'}) = \lambda W_{\alpha\beta}(q_0, \mathbf{k}), \tag{9}$$

for the SU(2) theory, where $W_{\alpha\beta}(q_0, \mathbf{k})$ is the eigenvector associated with the SU(2) isospin channels α and $\beta(\alpha = 1, 2$ and $\beta = 1, 2$) and λ, the corresponding eigenvalue. The divergence of the eigenvalue at a given energy q_0 signifies the pole of the t-matrix at the energy. The value of pole q_0 corresponds to the energy of holon pair corresponding to the eigenvector $W(q_0, \mathbf{k})$.

For each eigenvector we compute the contribution of the s-, p_x-, p_y- and $d_{x^2-y^2}$-orbitals. The orbital symmetry is determined from numerical fitting to the computed result of the eigenvector,

$$W(q_0, \mathbf{k}) = a_s(\cos k_x + \cos k_y) + a_{p_x} \sin k_x + a_{p_y} \sin k_y + a_d(\cos k_x - \cos k_y) \tag{10}$$

for the U(1) theory and

$$W_{\alpha\beta}(q_0, \mathbf{k}) = a_{s,\alpha\beta}(\cos k_x + \cos k_y) + a_{p_x,\alpha\beta} \sin k_x + a_{p_y,\alpha\beta} \sin k_y + a_{d,\alpha\beta}(\cos k_x - \cos k_y) \tag{11}$$

for the SU(2) theory. Here, a_l represents the weight of the l-th partial wave of holon pair in the U(1) theory with orbital angular momentum of $l = s$, p_x, p_y and d. For the SU(2) theory, $a_{l,\alpha\beta}$ represents the weight of the l-th partial wave in the b_α and b_β holon pairing channel with $\alpha, \beta = 1$, 2. The poles and eigenvectors are obtained for three different hole concentrations : underdoped, optimally doped and overdoped cases with $J/t = 0.3$ and $N = 10 \times 10$ square lattice.

III. RESULTS

In Fig. 1, we summarize our results for the energy poles and corresponding eigenvectors of the t-matrix. Above the on-set temperature T_c, there is no phase coherent holon pair bound state. As temperature is lowered to a temperature below T_c from a temperature above T_c, the lowest energy pole changes from a positive to negative sign, that is, a phase coherent bound state with negative energy occurs [14]. The predicted bound state has the s-wave pairing symmetry. As shown in Fig. 1, the energies of the higher lying pairing states remains to be positive (unbound) even below T_c.

In Table 1, we show the weight of each orbital for the eigenvector of the U(1) t-matrix at the lowest energy at temperature slightly above the on-set temperature ($T = T_c + 10^{-3}t$ with t, the hoping integral). The eigenvector of the lowest energy corresponds to the stable channel of holon pairing below T_c [14]. As shown in Table 1, stable holon pairing occurs in the pure s-wave channel. The contributions of the higher partial wave (p- and d-wave) symmetries virtually vanishes; they are found to be less than 10^{-13} times smaller than that of the s-wave contribution. The lowest lying holon pairing state remains to be the pure s-wave (having no mixing with other orbitals) with decreasing temperature from a temperature above T_c. It is now clear from this prediction that the Cooper pairing occurs with the pure $d_{x^2-y^2}$ symmetry resulting from the composite nature of the $d_{x^2-y^2}$-wave symmetry of spinon pairing and the s-wave symmetry of the holon pairing [22].

With the SU(2) t-matrix, there exist two eigenvectors corresponding to two bosons b_1 and b_2. The weight of each orbital for the first eigenvector is shown in Table 2 and the second, in Table 3. As shown in Table 2, the first eigenvector is of pure s-wave pairing symmetry with no phase difference between the b_1-b_1 and b_2-b_2 pair scattering channel. As shown in Table 3, the second eigenvector shows a dominance of the s-wave pairing symmetry with the phase difference of π between the b_1-b_1 and b_2-b_2 channels and a negligibly small contribution from the d-wave symmetry in the b_1-b_2 channel. The weights of other orbital contributions are negligibly small as shown in the Tables. It can be readily checked that only the second eigenvector in Table 3 leads to non-vanishing Cooper pair order parameter, while the first one in Table 2 results in vanishing Cooper pair order parameter [23]. The SU(2) theory differs from the U(1) theory in that the phase fluctuations of order parameters are taken

into account by allowing the b_2-boson [21]. There are two eigenvectors of t-matrix associated with the same energy pole in SU(2). However, there is only one stable s-wave channel of holon pairing with no other orbital mixing which leads to the non-vanishing Cooper pair order parameter for both the U(1) and SU(2) theories. It is now clear that the composition of the s-wave channel of the holon pair and the d-wave channel of the spinon pair leads to the pure $d_{x^2-y^2}$-wave symmetry of the Cooper pair order parameter [1]. Thus there exists no time-reversal symmetry breaking allowing no complex orbital mixing.

For the higher energy poles, the s-, p_x-, p_y- and $d_{x^2-y^2}$-wave states are predicted to occur as shown in Tables 4 and 5. Interestingly, the eigenvectors occur in a sequence of certain pattern as the energy pole of the t-matrix increases both above and below T_c as shown in Fig. 1 : s-wave state occurs in the lowest energy; the next four high lying energy states are of the s-, p_x-, p_y- and $d_{x^2-y^2}$-wave symmetries with nearly the same energy (that is, near degenerate); the next higher lying energy states are of s-, p_x- and p_y-wave symmetries with nearly the same energy and so on. Below we will discuss how the above sequential pattern arises in association with momenta available in the intermediate scattering states of holon pair .

First we analyze the locations of the poles. The poles of the t-matrix occur at discrete energies for both bound and unbound states of the holon pairs for finite lattice. With increasing lattice size ($L \to \infty$), the unbound states become continuum as the allowed momenta become continuum. The energy of holon pair with momenta ($\mathbf{k}$ and $-\mathbf{k}$) is the sum of kinetic energy and interaction energy. The magnitude of the interaction energy ($V \sim J|\Delta_f|^2$) is relatively small as compared to that of the kinetic energy ($K \sim t\chi$) with the ratio of $V/K \leq 0.1$. Thus the kinetic energy dominantly determines the energy of holon pair. Indeed, the predicted locations of poles in Tables 4 and 5 are close to the kinetic energy of holon pair with momenta $\mathbf{k} = (0,0), (2\pi/L, 0), (2\pi/L, 2\pi/L)$ and $(2\pi/L, 0)$ [24].

Here we pay attention to the intermediate scattering states of holon pair. This will allow us to examine how an initial holon pair state with an l-th partial wave states ($l = s, p_x, p_y$ or d) can be scattered into intermediate states having momenta $\mathbf{k}_1$ and $-\mathbf{k}_1$. The transition amplitude from the initial state to the intermediate state is, to first order,

$$T_{l \to \{\mathbf{k}_1, -\mathbf{k}_1\}} = \sum_{\mathbf{k}} v(\mathbf{k}_1 - \mathbf{k}) w_l(\mathbf{k})$$

$$= -J|\Delta_f|^2 \sum_{\mathbf{k}} \left[\cos(k_{1x} - k_x) + \cos(k_{1y} - k_y)\right] w_l(\mathbf{k}), \tag{12}$$

where $w_l(\mathbf{k})$ is the eigenvector with an l-th partial wave (orbital). The Feynmann diagram for this process is displayed in Fig. 2.

For the initial s-wave pairing state $w_l(\mathbf{k}) = (\cos k_x + \cos k_y)$, the transition amplitude is obtained to be

$$\begin{aligned} T_{s\to\{\mathbf{k}_1,-\mathbf{k}_1\}} &= -J|\Delta_f|^2 \sum_{\mathbf{k}} \left[\cos(k_{1x} - k_x) + \cos(k_{1y} - k_y)\right] (\cos k_x + \cos k_y) \\ &= -\frac{N}{2} J|\Delta_f|^2 (\cos k_{1x} + \cos k_{1y}). \end{aligned} \tag{13}$$

This amplitude does not vanish as long as the momentum of the intermediate state ($\mathbf{k_1}$) satisfies the condition of $(\cos k_{1x} + \cos k_{1y}) \neq 0$. In other words, the s-wave state has a non-vanishing transition amplitude to the intermediate state with momenta $\mathbf{k}_1$ and $-\mathbf{k}_1$ if $(\cos k_{1x} + \cos k_{1y}) \neq 0$. Thus, the s-wave scattering channel allows poles at the intermediate state energies of holon pairs with momenta $\mathbf{k}_1$ and $-\mathbf{k}_1$ as long as $(\cos k_{1x} + \cos k_{1y}) \neq 0$. For example, the s-wave eigenvector occurs at poles near $2\epsilon(0,0)$, $2\epsilon(2\pi/L,0)$, $2\epsilon(2\pi/L,2\pi/L)$ and $2\epsilon(4\pi/L,0)$ as shown in Table 4. This is because $(\cos k_{1x} + \cos k_{1y})$ does not vanish for momenta, $(0,0)$, $(2\pi/L,0)$, $(2\pi/L,2\pi/L)$ and $(4\pi/L,0)$. Similarly, one can readily understand why p- and d-wave scattering states (channels) occur only at certain energy levels [25]. For higher angular momentum channels such as $w_l(k) = \cos 2k_x$ or $w_l(k) = \sin 2k_x$, the transition amplitude in Eq. (13) identically vanishes. Therefore, t-matrix can not allow poles in the scattering channels corresponding to these higher orbital angular momentum states; only the s, p_x, p_y and $d_{x^2-y^2}$ state can occur.

The above analysis with the U(1) theory is readily applied to the SU(2) case to find the location of poles and the corresponding eigenvectors. The only difference is that in SU(2) the number of eigenvectors for each pole is doubled owing to the presence of two degenerate bosons b_1 and b_2. These two eigenvectors are found to be degenerate. One is the eigenvector with no phase difference between the $b_1 - b_1$ and $b_2 - b_2$ pairing channel. The other is the eigenvector with the phase difference of π. Interestingly, there is a small component of $b_1 - b_2$ boson pairing for the case of the latter. The weight of this off-diagonal pairing component is negligibly small in the lowest energy pole (e.g. $\sim 10^{-4}$ for $\omega = 0.0097$ with $x = 0.07$), but become appreciably large at higher energy poles (e.g. ~ 0.2 for $\omega/t = 1.1236$ with

$x = 0.07$). On the other hand, the weight of the $b_1 - b_2$ boson pairing state decreases with the increase of hole concentration as shown in Table 5. This is due to the decreasing trend of the spinon pairing order parameter Δ_f with increasing hole concentration. It is of note that the off-diagonal element between b_1 and b_2 bosons (or vice versa) in the Hamiltonian is proportional to the spinon pairing order parameter in Eq.(3).

It is also noted that there occurs not only s- and d- wave states, but also p- wave state for holon pairing. The p- wave state is not allowed because it has the odd value of the orbital angular momentum and violate the symmetric waves. With the inclusion of exchange channel only s- and d- wave states appear in association with eigenvectors in the t-matrix as shown in Fig. 3.

IV. SUMMARY

In the present study, we investigated the orbital contributions to both U(1) and SU(2) t-matrices both above and below T_c and examined the lowest energy and higher energy poles respectively by applying the Bethe-Salpeter equation to the U(1) and SU(2) holon-pair boson theory. We showed that at the lowest energy (below T_c) there is only one holon pairing bound state which gives rise to the non-vanishing Cooper pair order parameter for both the U(1) and SU(2) theories. We found that the holon pairing at and below T_c is made of the pure s-wave with no complex orbital mixing and thus the Cooper pair of d-wave symmetry as a composite of the s-wave holon pair and the d-wave spinon pair. As a consequence there exits no time reversal symmetry breaking. Thus we argue that symmetry breaking is caused by a different origin since the complex orbital mixing such as $d_{x^2-y^2} + id_{xy}$ and $d_{x^2-y^2} + is$ did not appear.

APPENDIX A: DERIVATION OF THE U(1) HOLON HAMILTONIAN

The t-J Hamiltonian of interest is given by,

$$H = -t \sum_{<i,j>} (c^\dagger_{i\sigma} c_{j\sigma} + c.c.) + J \sum_{<i,j>} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j)$$
$$-\mu \sum_i c^\dagger_{i\sigma} c_{i\sigma} \quad \text{(A1)}$$

and the Heisenberg interaction term is rewritten

$$H_J = J \sum_{<i,j>} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j)$$
$$= -\frac{J}{2} \sum_{<i,j>} (c^\dagger_{i\downarrow} c^\dagger_{j\uparrow} - c^\dagger_{i\uparrow} c^\dagger_{j\downarrow})(c_{j\uparrow} c_{i\downarrow} - c_{j\downarrow} c_{i\uparrow}). \quad \text{(A2)}$$

Here t is the hopping energy and $\mathbf{S}_i$, the electron spin operator at site i, $\mathbf{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$ with $\boldsymbol{\sigma}_{\alpha\beta}$, the Pauli spin matrix element. n_i is the electron number operator at site i, $n_i = c^\dagger_{i\sigma} c_{i\sigma}$. μ is the chemical potential.

In the U(1) slave-boson representation [17–20], with single occupancy constraint at site i the electron annihilation operator $c_{i\sigma}$ is taken as a composite operator of the spinon (neutrally charged fermion) annihilation operator $f_{i\sigma}$ and the holon (positively charged boson) creation operator $b^\dagger_i$, and thus, $c_{i\sigma} = f_{i\sigma} b^\dagger_i$. Rigorously speaking, it should be noted that the expression $c_{i\sigma} = b^\dagger_i f_{i\sigma}$ is not precise since these operators belong to different Hilbert spaces and thus the equality sign here should be taken only as a symbol for mapping. Using $c_{i\sigma} = f_{i\sigma} b^\dagger_i$ and introducing the Lagrange multiplier term (the last term in Eq.(A3)) to enforce single occupancy constraint, the t-J Hamiltonian is rewritten,

$$H = -t \sum_{<i,j>} \Big((f^\dagger_{i\sigma} b_i)(b^\dagger_j f_{j\sigma}) + c.c. \Big)$$
$$-\frac{J}{2} \sum_{<i,j>} b_i b_j b^\dagger_j b^\dagger_i (f^\dagger_{i\downarrow} f^\dagger_{j\uparrow} - f^\dagger_{i\uparrow} f^\dagger_{j\downarrow})(f_{j\uparrow} f_{i\downarrow} - f_{j\downarrow} f_{i\uparrow})$$
$$-\mu \sum_i f^\dagger_{i\sigma} b_i f_{i\sigma} b^\dagger_i - i \sum_i \lambda_i (b^\dagger_i b_i + f^\dagger_{i\sigma} f_{i\sigma} - 1). \quad \text{(A3)}$$

The coupling between physical quantities A and B is decomposed into terms involving fluctuations of A, i.e., $(A- < A >)$ and B, i.e., $(B- < B >)$, separately uncorrelated mean field contribution of $< A >$ and $< B >$ and correlation between fluctuations of A and B, that is, $(A- < A >)$ and $(B- < B >)$ respectively; $AB =< B > A+ < A > B- < A ><$

$B >+(A-<A>)(B-<B>)$. Setting $A=b_i b_j b_j^\dagger b_i^\dagger$ for charge (holon) contribution and $B=(f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})$ for spin (spinon) contribution, the Heisenberg coupling term can be decomposed into terms involving coupling between the charge and spin fluctuations separately, the mean field contributions and coupling (correlation) between fluctuations (charge and spin fluctuations). Using such decomposition of the Heisenberg interaction term for Eq.(A3), we write the U(1) Hamiltonian

$$
\begin{aligned}
H_{t-J}^{U(1)} = -t \sum_{<i,j>} & (f_{i\sigma}^\dagger f_{j\sigma} b_j^\dagger b_i + c.c.) \\
& -\frac{J}{2}\sum_{<i,j>}\Big[\Big\langle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})\Big\rangle b_i b_j b_j^\dagger b_i^\dagger \\
& +\Big\langle b_i b_j b_j^\dagger b_i^\dagger \Big\rangle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow}) \\
& -\Big\langle b_i b_j b_j^\dagger b_i^\dagger \Big\rangle \Big\langle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})\Big\rangle \\
& +\Big(b_i b_j b_j^\dagger b_i^\dagger - \Big\langle b_i b_j b_j^\dagger b_i^\dagger \Big\rangle \Big) \\
& \times\Big((f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow}) - \Big\langle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})\Big\rangle\Big)\Big] \\
& -\mu\sum_i f_{i\sigma}^\dagger f_{i\sigma}(1+b_i^\dagger b_i) - i\sum_i \lambda_i (f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i - 1). \qquad \text{(A4)}
\end{aligned}
$$

Noting that $[b_i, b_j^\dagger] = \delta_{ij}$ for boson, the intersite charge (holon) interaction term (the second term) in Eq.(A4) is rewritten,

$$
\begin{aligned}
& -\frac{J}{2}\Big\langle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})\Big\rangle b_i b_j b_j^\dagger b_i^\dagger \\
& = -\frac{J}{2} < |\Delta_{ij}^f| >^2 \Big(1 + b_i^\dagger b_i + b_j^\dagger b_j + b_i^\dagger b_j^\dagger b_j b_i\Big), \qquad \text{(A5)}
\end{aligned}
$$

with $\Delta_{ij}^f = f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow}$, the spinon pairing field. The third term in Eq.(A4) represents the intersite spin (spinon) interaction and is rewritten,

$$
\begin{aligned}
& -\frac{J}{2}\Big\langle b_i b_j b_j^\dagger b_i^\dagger \Big\rangle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow}) \\
& = -\frac{J_p}{2}(f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow}), \qquad \text{(A6)}
\end{aligned}
$$

where $J_p = J(1+<b_i^\dagger b_i> + <b_j^\dagger b_j> + <b_i^\dagger b_i b_j^\dagger b_j>)$ or $J_p = J(1-x)^2$ with x, the uniform hole doping concentration. The fourth term in Eq.(A4) is written,

$$
\begin{aligned}
& \frac{J}{2}\Big\langle b_i b_j b_j^\dagger b_i^\dagger \Big\rangle \Big\langle (f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger)(f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow})\Big\rangle \\
& = \frac{J}{2}\Big(1+ <b_i^\dagger b_i> + <b_j^\dagger b_j> + <b_i^\dagger b_i b_j^\dagger b_j>\Big) < |\Delta_{ij}^f|^2 > .
\end{aligned}
$$

(A7)

The intersite spinon interaction term in Eq.(A6) is decomposed into the direct, exchange and pairing channels [20],

$$
\begin{aligned}
&-\frac{J_p}{2}(f^\dagger_{i\downarrow}f^\dagger_{j\uparrow}-f^\dagger_{i\uparrow}f^\dagger_{j\downarrow})(f_{j\uparrow}f_{i\downarrow}-f_{j\downarrow}f_{i\uparrow})\\
&=\frac{J_p}{4}\Big[\sum_{k=1}^{3}(f^\dagger_{i\alpha}\sigma^k_{\alpha\beta}f_{i\beta})(f^\dagger_{j\gamma}\sigma^k_{\gamma\delta}f_{j\delta})-(f^\dagger_{i\alpha}\sigma^0_{\alpha\beta}f_{i\beta})(f^\dagger_{i\gamma}\sigma^0_{\gamma\delta}f_{j\delta})\Big]\\
&=v_D+v_E+v_P
\end{aligned}
\tag{A8}
$$

with $\sigma^0=I$, the identity matrix and $\sigma^{1,2,3}$, the Pauli spin matrices, where v_D, v_E and v_P are the spinon interaction terms of the direct, exchange and pairing channels respectively [15],

$$v_D=-\frac{J_p}{8}\sum_{k=0}^{3}(f^\dagger_i\sigma^k f_i)(f^\dagger_j\sigma^k f_j), \tag{A9}$$

$$v_E=-\frac{J_p}{4}\Big((f^\dagger_{i\sigma}f_{j\sigma})(f^\dagger_{j\sigma}f_{i\sigma})-n_i\Big), \tag{A10}$$

$$v_P=-\frac{J_p}{2}(f^\dagger_{i\downarrow}f^\dagger_{j\uparrow}-f^\dagger_{i\uparrow}f^\dagger_{j\downarrow})(f_{j\uparrow}f_{i\downarrow}-f_{j\downarrow}f_{i\uparrow}). \tag{A11}$$

Here σ^0 is the unit matrix and $\sigma^{1,2,3}$, the Pauli spin matrices.

Combining Eq.(A5) and Eq.(A7), we have

$$
\begin{aligned}
&-\frac{J}{2}<|\Delta^f_{ij}|^2>\Big(1+b^\dagger_ib_i+b^\dagger_jb_j+b^\dagger_ib^\dagger_jb_jb_i\Big)\\
&+\frac{J}{2}<|\Delta^f_{ij}|^2>\Big(1+<b^\dagger_ib_i>+<b^\dagger_jb_j>+<b^\dagger_ib_ib^\dagger_jb_j>\Big)\\
&=-\frac{J}{2}<|\Delta^f_{ij}|^2>b^\dagger_ib^\dagger_jb_jb_i+\frac{J}{2}<|\Delta^f_{ij}|^2><b^\dagger_ib_ib^\dagger_jb_j>\\
&-\frac{J}{2}<|\Delta^f_{ij}|^2>\Big[\Big(b^\dagger_ib_i-<b^\dagger_ib_i>\Big)+\Big(b^\dagger_jb_j-<b^\dagger_jb_j>\Big)\Big].
\end{aligned}
\tag{A12}
$$

Collecting the decomposed terms Eq.(A5) through Eq.(A7) in association with Eqs.(A8) through Eq.(A12), we write

$$
\begin{aligned}
H_J=&-\frac{J}{2}\sum_{<i,j>}|\Delta^f_{ij}|^2b^\dagger_ib^\dagger_jb_jb_i\\
&-J_p\sum_{<i,j>}\Big[\frac{1}{2}(f^\dagger_{i\downarrow}f^\dagger_{j\uparrow}-f^\dagger_{i\uparrow}f^\dagger_{j\downarrow})(f_{j\uparrow}f_{i\downarrow}-f_{j\downarrow}f_{i\uparrow})\\
&+\frac{1}{4}\Big((f^\dagger_{i\sigma}f_{j\sigma})(f^\dagger_{j\sigma}f_{i\sigma})-n_i\Big)\\
&+\frac{1}{8}\sum_{k=0}^{3}(f^\dagger_i\sigma^kf_i)(f^\dagger_j\sigma^kf_j)\Big]
\end{aligned}
$$

$$+\frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2 < b_i^\dagger b_i >< b_j^\dagger b_j >$$

$$-\frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2\left[\left(b_i^\dagger b_i - < b_i^\dagger b_i >\right) + \left(b_j^\dagger b_j - < b_j^\dagger b_j >\right)\right], \quad \text{(A13)}$$

where we considered $< |\Delta_{ij}^f|^2 >= |\Delta_{ij}^f|^2$ and ignored the fifth term in Eq.(A4).

As are shown in Eqs.(A9) through (A11) the spinon interaction term is decomposed into the direct, exchange and pairing channels respectively. Proper Hubbard-Stratonovich transformations corresponding to these channels and saddle point approximation leads to the effective Hamiltonian,

$$\begin{aligned}
H_{eff} = &\frac{J_p}{4}\sum_{<i,j>}\left[|\chi_{ij}|^2 - \chi_{ij}^*(f_{i\sigma}^\dagger f_{j\sigma} + \frac{4t}{J_p}b_i^\dagger b_j) - c.c.\right] \\
&-\frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2 b_j^* b_i^* b_j b_i \\
&+\frac{J_p}{2}\sum_{<i,j>}\left[|\Delta_{ij}^f|^2 - \Delta_{ij}^f(f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger - f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger) - c.c.\right] \\
&+\frac{J_p}{2}\sum_{<i,j>}\sum_{l=0}^{3}\left[(\rho_j^l)^2 - \rho_j^l(f_i^\dagger\sigma^l f_i)\right] + \frac{J_p}{2}\sum_i(f_{i\sigma}^\dagger f_{i\sigma}) \\
&+\frac{4t^2}{J_p}\sum_{<i,j>}(b_i^\dagger b_j)(b_j^\dagger b_i) \\
&+\frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2 < b_i^\dagger b_i >< b_j^\dagger b_j > \\
&-\frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2\left[\left(b_i^\dagger b_i - < b_i^\dagger b_i >\right) + \left(b_j^\dagger b_j - < b_j^\dagger b_j >\right)\right] \\
&-\mu\sum_i f_{i\sigma}^\dagger f_{i\sigma}(1 + b_i^\dagger b_i) - i\sum_i\lambda_i(f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i - 1), \quad \text{(A14)}
\end{aligned}$$

where $\Delta_{ij}^b =< b_i b_j >$, $\chi_{ij} =< f_{i\sigma}^\dagger f_{j\sigma} + \frac{4t}{J_p}b_i^\dagger b_j >$, $\Delta_{ij}^f =< f_{j\uparrow}f_{i\downarrow} - f_{j\downarrow}f_{i\uparrow} >$ and $\rho_i^k =< \frac{1}{2}f_i^\dagger\sigma^k f_i >$ are proper saddle points.

We note that $\rho_i^l = \frac{1}{2} < f_i^\dagger\sigma^l f_i >=< S_i^l >= 0$ for $l = 1, 2, 3$, $\rho_i^0 = \frac{1}{2} < f_{i\sigma}^\dagger f_{i\sigma} >= \frac{1}{2}$ for $l = 0$ for the contribution of the direct spinon interaction term (the fourth term). The expression $(b_j^\dagger b_i)(b_i^\dagger b_j)$ in the fifth term of Eq.(A14) represents the exchange interaction channel. The exchange channel will be ignored owing to a large cost in energy, $U \approx \frac{4t^2}{J}$ [20] [21]. The resulting effective Hamiltonian is

$$H^{MF} = H^{\Delta,\chi} + H^b + H^f, \quad \text{(A15)}$$

where $H^{\Delta,\chi}$ represents the the saddle point energy involved with the spinon pairing order parameter Δ^f and the hopping order parameter χ,

$$H^{\Delta,\chi} = J \sum_{<i,j>} \frac{1}{2}|\Delta^f_{ij}|^2 x^2 + \frac{J_p}{2} \sum_{<i,j>} \Big[|\Delta^f_{ij}|^2 + \frac{1}{2}|\chi_{ij}|^2 + \frac{1}{4}\Big], \tag{A16}$$

H^b is the holon Hamiltonian,

$$\begin{aligned} H^b = &-t \sum_{<i,j>} \Big[\chi^*_{ij}(b^\dagger_i b_j) + c.c.\Big] \\ &- \sum_{<i,j>} \frac{J}{2}|\Delta^f_{ij}|^2 b^\dagger_i b^\dagger_j b_j b_i \\ &- \sum_i \mu^b_i (b^\dagger_i b_i - x), \end{aligned} \tag{A17}$$

where $\mu^b_i = i\lambda_i + \frac{J}{2}\sum_{j=i\pm\hat{x},i\pm\hat{y}} |\Delta^f_{ij}|^2$ and H^f, the spinon Hamiltonian,

$$\begin{aligned} H^f = &-\frac{J_p}{4} \sum_{<i,j>} \Big[\chi^*_{ij}(f^\dagger_{i\sigma} f_{j\sigma}) + c.c.\Big] \\ &- \frac{J_p}{2} \sum_{<i,j>} \Big[\Delta^{f*}_{ij}(f_{j\uparrow} f_{i\downarrow} - f_{j\downarrow} f_{i\uparrow}) + c.c.\Big] \\ &- \sum_i \mu^f_i \left(f^\dagger_{i\sigma} f_{i\sigma} - (1-x)\right), \end{aligned} \tag{A18}$$

where $\mu^f_i = \mu(1-x) + i\lambda_i$.

APPENDIX B: DERIVATION OF THE SU(2) HOLON HAMILTONIAN

The SU(2) slave-boson representation of the above t-J Hamiltonian reads [15]

$$\begin{aligned} H = &-\frac{t}{2} \sum_{<i,j>\sigma} \Big[(f^\dagger_{\sigma i} f_{\sigma j})(b^\dagger_{1j} b_{1i} - b^\dagger_{2i} b_{2j}) + c.c. \\ &+(f_{2i} f_{1j} - f_{1i} f_{2j})(b^\dagger_{1j} b_{2i} + b^\dagger_{1i} b_{2j}) + c.c.\Big] \\ &-\frac{J}{2} \sum_{<i,j>} (1 + h^\dagger_i h_i)(1 + h^\dagger_j h_j) \times \\ &(f^\dagger_{2i} f^\dagger_{1j} - f^\dagger_{1i} f^\dagger_{2j})(f_{1j} f_{2i} - f_{2j} f_{1i}) - \mu_0 \sum_i h^\dagger_i h_i \\ &- \sum_i \Big[i\lambda^{(1)}_i (f^\dagger_{1i} f^\dagger_{2i} + b^\dagger_{1i} b_{2i}) + i\lambda^{(2)}_i (f_{2i} f_{1i} + b^\dagger_{2i} b_{1i}) \\ &+ i\lambda^{(3)}_i (f^\dagger_{1i} f_{1i} - f_{2i} f^\dagger_{2i} + b^\dagger_{1i} b_{1i} - b^\dagger_{2i} b_{2i})\Big], \end{aligned} \tag{B1}$$

where $\lambda^{(1),(2),(3)}_i$ are the real Lagrangian multipliers to enforce the local single occupancy constraint in the SU(2) slave-boson representation [21]. Taking decomposition of the Heisenberg

interaction term above into terms involving charge and spin fluctuations separately, uncorrelated mean field contributions and correlated fluctuations, i.e., correlations between charge and spin fluctuations as in the U(1) case, the SU(2) Hamiltonian is rewritten,

$$
\begin{aligned}
H_{t-J}^{SU(2)} = &-\frac{t}{2}\sum_{<i,j>}\Big[(f_{i\alpha}^{\dagger}f_{j\alpha})(b_{j1}^{\dagger}b_{i1}-b_{i2}^{\dagger}b_{j2})+c.c.\\
&+(f_{i2}f_{j1}-f_{i1}f_{j2})(b_{j1}^{\dagger}b_{i2}+b_{i1}^{\dagger}b_{j2})+c.c.\Big]\\
&-\frac{J}{2}\sum_{<i,j>}\Big[\Big\langle(f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})\Big\rangle(1+h_i^{\dagger}h_i)(1+h_j^{\dagger}h_j)\\
&+\Big\langle(1+h_i^{\dagger}h_i)(1+h_j^{\dagger}h_j)\Big\rangle(f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})\\
&-\Big\langle(1+h_i^{\dagger}h_i)(1+h_j^{\dagger}h_j)\Big\rangle\Big\langle(f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})\Big\rangle\\
&+\Big((1+h_i^{\dagger}h_i)(1+h_j^{\dagger}h_j)-\Big\langle(1+h_i^{\dagger}h_i)(1+h_j^{\dagger}h_j)\Big\rangle\Big)\times\\
&\Big((f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})-\Big\langle(f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})\Big\rangle\Big)\Big]\\
&-\mu\sum_i(1-h_i^{\dagger}h_i)\\
&-\sum_i\Big(i\lambda_i^{(1)}(f_{i1}^{\dagger}f_{i2}^{\dagger}+b_{i1}^{\dagger}b_{i2})+i\lambda_i^{(2)}(f_{i2}f_{i1}+b_{i2}^{\dagger}b_{i1})\\
&+i\lambda_i^{(3)}(f_{i1}^{\dagger}f_{i1}-f_{i2}f_{i2}^{\dagger}+b_{i1}^{\dagger}b_{i1}-b_{i2}^{\dagger}b_{i2})\Big).
\end{aligned}
\tag{B2}
$$

The intersite holon interaction (the third term in Eq.(B2)) is rewritten,

$$
\begin{aligned}
&-\frac{J}{2}\Big\langle(f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})\Big\rangle(1+h_i^{\dagger}h_i)(1+h_j^{\dagger}h_j)\\
&=-\frac{J}{2}<|\Delta_{ij}^f|^2>(1+h_i^{\dagger}h_i+h_j^{\dagger}h_j+h_i^{\dagger}h_ih_j^{\dagger}h_j)\\
&\approx-\frac{J}{2}|\Delta_{ij}^f|^2(1+b_{i\alpha}^{\dagger}b_{i\alpha}+b_{j\alpha}^{\dagger}b_{j\alpha}+b_{i\alpha}^{\dagger}b_{j\beta}^{\dagger}b_{j\beta}b_{i\alpha})
\end{aligned}
\tag{B3}
$$

where $\Delta_{ij}^f=\Big\langle f_{j1}f_{i2}-f_{j2}f_{i1}\Big\rangle$ is the spinon singlet pairing order parameter. The intersite spinon interaction (the fourth term in Eq.(B2)) is rewritten in terms of decomposed Hatree-Fock-Bogoliubov channels in the same way as in the U(1) case,

$$
-\frac{J_p}{2}(f_{i2}^{\dagger}f_{j1}^{\dagger}-f_{i1}^{\dagger}f_{j2}^{\dagger})(f_{j1}f_{i2}-f_{j2}f_{i1})=v_D+v_E+v_P, \tag{B4}
$$

where v_D, v_E and v_P are the interactions corresponding to the direct, exchange and pairing channels respectively,

$$v_D = -\frac{J_p}{8}\sum_{l=0}^{3}(f_i^\dagger \sigma^l f_i)(f_j^\dagger \sigma f_j), \tag{B5}$$

$$v_E = -\frac{J_p}{4}\Big((f_{i\sigma}^\dagger f_{j\sigma})(f_{j\sigma}^\dagger f_{i\sigma}) - n_i\Big), \tag{B6}$$

$$v_P = -\frac{J_p}{2}(f_{i2}^\dagger f_{j1}^\dagger - f_{i1}^\dagger f_{j2}^\dagger)(f_{j1}f_{i2} - f_{j2}f_{i1}). \tag{B7}$$

Here σ^0 is the unit matrix and $\sigma^{1,2,3}$, the Pauli spin matrices. The fifth term in Eq.(B2) is written,

$$\begin{aligned}
&\frac{J}{2}\Big\langle(1+h_i^\dagger h_i)(1+h_j^\dagger h_j)\Big\rangle \times \\
&\Big\langle(f_{i2}^\dagger f_{j1}^\dagger - f_{i1}^\dagger f_{j2}^\dagger)(f_{j1}f_{i2} - f_{j2}f_{i1})\Big\rangle \\
\approx\ &\frac{J}{2}\Big\langle(1+h_i^\dagger h_i)\Big\rangle\Big\langle(1+h_j^\dagger h_j)\Big\rangle \times \\
&\Big\langle(f_{i2}^\dagger f_{j1}^\dagger - f_{i1}^\dagger f_{j2}^\dagger)\Big\rangle\Big\langle(f_{j1}f_{i2} - f_{j2}f_{i1})\Big\rangle \\
=\ &\frac{J}{2}|\Delta_{ij}^f|^2(1+ < h_i^\dagger h_i > + < h_j^\dagger h_j > + < h_i^\dagger h_i >< h_j^\dagger h_j >).
\end{aligned} \tag{B8}$$

We introduced $\Big\langle(f_{i2}^\dagger f_{j1}^\dagger - f_{i1}^\dagger f_{j2}^\dagger)(f_{j1}f_{i2} - f_{j2}f_{i1})\Big\rangle \approx \Big\langle(f_{i2}^\dagger f_{j1}^\dagger - f_{i1}^\dagger f_{j2}^\dagger)\Big\rangle\Big\langle(f_{j1}f_{i2} - f_{j2}f_{i1})\Big\rangle = |\Delta_{ij}^f|^2$ and $\Big\langle(1+h_i^\dagger h_i)(1+h_j^\dagger h_j)\Big\rangle \approx \Big\langle(1+h_i^\dagger h_i)\Big\rangle\Big\langle(1+h_j^\dagger h_j)\Big\rangle$.

By introducing the Hubbard-Stratonovich fields, ρ_i^k, χ_{ij} and Δ_{ij} for the spinon direct, exchange and pairing order shown in Eqs.(B5), (B6), (B7), we rewrite the effective Hamiltonian for Eq.(B2),

$$\begin{aligned}
H_{SU(2)}^{MF} &= \frac{J_p}{4}\sum_{<i,j>}\Big[|\chi_{ij}|^2 - \chi_{ij}^*\{f_{i\sigma}^\dagger f_{\sigma j} + \frac{2t}{J_p}(b_{i1}^\dagger b_{j1} - b_{j2}^\dagger b_{2i})\} - c.c.\Big] \\
&- \frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2\Big[b_{i\alpha}^\dagger b_{j\beta}^\dagger b_{j\alpha}b_{i\beta}\Big] \\
&+ \frac{J_p}{2}\sum_{<i,j>}\Big[|\Delta_{ij}|^2 - \Delta_{ij}\{(f_{i2}^\dagger f_{j1}^\dagger - f_{i1}^\dagger f_{j2}^\dagger) - \frac{t}{J_p}(b_{j1}^\dagger b_{i2} + b_{i1}^\dagger b_{j2})\} - c.c.\Big] \\
&+ \frac{J_p}{2}\sum_{<i,j>}\sum_{l=0}^{3}\Big((\rho_{ij}^l)^2 - \rho_{ij}^l(f_i^\dagger \sigma^l f_i)\Big) \\
&+ \frac{t^2}{J_p}\sum_{<i,j>}\Big[(b_{i1}^\dagger b_{j1} - b_{j2}^\dagger b_{i2})(b_{j1}^\dagger b_{i1} - b_{i2}^\dagger b_{j2}) \\
&\quad + \frac{1}{2}(b_{j1}^\dagger b_{i2} + b_{i1}^\dagger b_{j2})(b_{i2}^\dagger b_{j1} + b_{j2}^\dagger b_{i1})\Big] \\
&- \frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2\Big[h_j^\dagger h_j + h_i^\dagger h_i - < h_j^\dagger h_j > - < h_i^\dagger h_i >\Big] + \frac{J}{2}\sum_{<i,j>}|\Delta_{ij}^f|^2 x^2
\end{aligned}$$

$$
\begin{aligned}
&+\frac{J_p}{2}\sum_i (f_{i\sigma}^\dagger f_{i\sigma}) - \mu\sum_i (1-h_i^\dagger h_i) \\
&-\sum_i \Big[i\lambda_i^1 (f_{i1}^\dagger f_{i2}^\dagger + b_{i1}^\dagger b_{i2}) + i\lambda_i^2 (f_{i2} f_{i1} + b_{i2}^\dagger b_{i1}) \\
&\quad + i\lambda_i^3 (f_{i1}^\dagger f_{i1} - f_{i2} f_{i2}^\dagger + b_{i1}^\dagger b_{i1} - b_{i2}^\dagger b_{i2}) \Big],
\end{aligned}
\tag{B9}
$$

where $\chi_{ij} =< f_{i\sigma}^\dagger f_{j\sigma} + \frac{2t}{J_p}(b_{i1}^\dagger b_{j1} - b_{j2}^\dagger b_{i2}) >$, $\Delta_{ij} = \left\langle (f_{i1} f_{j2} - f_{i2} f_{j1}) - \frac{t}{J_p}(b_{i2}^\dagger b_{j1} + b_{j2}^\dagger b_{i1}) \right\rangle = \Delta_{ij}^f - \frac{t}{J(1-x)} \chi_{ij;12}^b$, with $\chi_{ij;12}^b = \left\langle b_{i2}^\dagger b_{j1} + b_{j2}^\dagger b_{i1} \right\rangle$ and x, the hole doping rate.

To simplify the effective Hamiltonian, we rearrange each term in Eq.(B9). For the paramagnetic state, we obtain $\rho_i^l = \frac{1}{2}(f_i^\dagger \sigma^l f_i) =< S_i^l >= 0$ for $l = 1, 2, 3$ and $\rho_i^0 = \frac{1}{2} < f_{i\sigma}^\dagger f_{i\sigma} >= \frac{1}{2}$ for $l = 0$. The one-body holon term (the sixth term) in the above Hamiltonian is incorporated into the effective chemical potential term. By setting $\Delta_{ij} = \Delta_{ij}^f - \frac{t}{J(1-x)}\chi_{ij;12}^b$ where $\Delta_{ij}^f =< f_{i1} f_{j2} - f_{i2} f_{j1} >$ and $\chi_{ij;12}^b = \left\langle b_{i2}^\dagger b_{j1} + b_{j2}^\dagger b_{i1} \right\rangle$, we rearrange the third term and the second term in the bracket of the fifth term to obtain the effective Hamiltonian,

$$
\begin{aligned}
H_{SU(2)}^{MF} &= \frac{J}{2}\sum_{<i,j>} |\Delta_{ij}^f|^2 x^2 + \frac{J_p}{2}\sum_{<i,j>} \Big[|\Delta_{ij}^f|^2 + \frac{1}{2}|\chi_{ij}|^2 + \frac{1}{4} \Big] \\
&-\frac{t}{2}\sum_{<i,j>} \Big[\chi_{ij}^* (b_{i1}^\dagger b_{j1} - b_{j2}^\dagger b_{i2}) - \Delta_{ij}^f (b_{j1}^\dagger b_{i2} + b_{i1}^\dagger b_{j2}) \Big] - c.c. \\
&-\frac{J_p}{4}\sum_{<i,j>} \Big[\chi_{ij}^* (f_{i\sigma}^\dagger f_{j\sigma}) + c.c. \Big] \\
-&\frac{J}{2}\sum_{<i,j>} |\Delta_{ij}^f|^2 \Big[b_{i\alpha}^\dagger b_{j\beta}^\dagger b_{j\alpha} b_{i\beta} \Big] \\
&-\frac{J_p}{2}\sum_{<i,j>} \Big[\Delta_{ij}^{f*} (f_{j1} f_{i2} - f_{j2} f_{i1}) + c.c. \Big] \\
&-\sum_i \mu_i^b (h_i^\dagger h_i - x) \\
&-\sum_i \Big[i\lambda_i^1 (f_{i1}^\dagger f_{i2}^\dagger + b_{i1}^\dagger b_{i2}^\dagger) + i\lambda_i^2 (f_{i2} f_{i1} + b_{i2} b_{i1}) \\
&+ i\lambda_i^3 (f_{i1}^\dagger f_{i1} - f_{i2} f_{i2}^\dagger + b_{i1}^\dagger b_{i1} - b_{i2}^\dagger b_{i2}) \Big] \\
&-\frac{t}{2}\sum_{<i,j>} \Big(\Delta_{ij}^f - (f_{j1} f_{i2} - f_{j2} f_{i1}) \Big) \chi_{ij;12}^{b*} - c.c. \\
&+\frac{t^2}{2J_p}\sum_{<i,j>} \Big| \chi_{ij;12}^b - (b_{i2}^\dagger b_{j1} + b_{j2}^\dagger b_{i1}) \Big|^2 \\
&+\frac{t^2}{J_p}\sum_{<i,j>} (b_{i1}^\dagger b_{j1} - b_{j2}^\dagger b_{i2})(b_{j1}^\dagger b_{i1} - b_{i2}^\dagger b_{j2}),
\end{aligned}
\tag{B10}
$$

where $\mu_i^b = -\mu + \frac{J}{2}\sum_{j=i\pm\hat{x}, i\pm\hat{y}} |\Delta_{ij}^f|^2$.

We neglect correlations between the fluctuations of order parameters. This is because correlations between the spin (spinon pair) and charge (holon pair) fluctuations are expected to be small as compared to the saddle point contribution of the order parameters (the first and second terms in the above Hamiltonian) particularly near the pseudogap temperature and the bose condensation temperature. However, individual fluctuations of the spinon pairing and holon pairing order parameters are not ignored. We also neglect the fluctuations of order parameters (ninth and tenth terms) in Eq.(B10). The ninth term represents the fluctuation of spinon pairing order parameter and we neglect it owing to its vanishment as an expectation value. The tenth term represents the fluctuations of holon hopping order parameter and is negligible at low temperature as its fluctuations die out in the low temperature regions where pairing order parameters Δ^f begins to open. Owing to the high energy cost involved with the Coulomb repulsion energy the exchange interaction terms (the last positive energy terms) will be ignored [20]- [21]. We then obtain the mean field Hamiltonian, $H = H^{\Delta,\chi} + H^b + H^f$, where $H^{\Delta,\chi}$ is the saddle point contribution of order parameters, χ and Δ^f,

$$H^{\Delta,\chi}_{SU(2)} = \frac{J}{2}\sum_{<i,j>} |\Delta^f_{ij}|^2 x^2 + \frac{J_p}{2}\sum_{<i,j>}\Big[|\Delta^f_{ij}|^2 + \frac{1}{2}|\chi_{ij}|^2 + \frac{1}{4}\Big], \tag{B11}$$

H^b is the holon Hamiltonian,

$$\begin{aligned} H^b_{SU(2)} &= -\frac{t}{2}\sum_{<i,j>}\Big[\chi^*_{ij}(b^\dagger_{i1}b_{j1} - b^\dagger_{j2}b_{i2}) \\ &-\Delta^f_{ij}(b^\dagger_{j1}b_{i2} + b^\dagger_{i1}b_{j2})\Big] - c.c. \\ -&\frac{J}{2}\sum_{<i,j>}|\Delta^f_{ij}|^2\Big[b^\dagger_{i\alpha}b^\dagger_{j\beta}b_{j\alpha}b_{i\beta}\Big] \\ &-\sum_i\Big[\mu^b_i(h^\dagger_i h_i - x) + i\lambda^1_i(b^\dagger_{i1}b^\dagger_{i2}) + i\lambda^2_i(b_{i2}b_{i1}) \\ &+i\lambda^3_i(b^\dagger_{i1}b_{i1} - b^\dagger_{i2}b_{i2})\Big], \end{aligned} \tag{B12}$$

and H^f, the spinon Hamiltonian,

$$\begin{aligned} H^f_{SU(2)} &= -\frac{J_p}{4}\sum_{<i,j>}\Big[\chi^*_{ij}(f^\dagger_{i\sigma}f_{j\sigma}) + c.c.\Big] \\ &-\frac{J_p}{2}\sum_{<i,j>}\Big[\Delta^{f*}_{ij}(f_{j1}f_{i2} - f_{j2}f_{i1}) + c.c.\Big], \\ &-\sum_i\Big[i\lambda^1_i(f^\dagger_{i1}f^\dagger_{i2}) + i\lambda^2_i(f_{i2}f_{i1}) + i\lambda^3_i(f^\dagger_{i1}f_{i1} - f_{i2}f^\dagger_{i2})\Big], \end{aligned} \tag{B13}$$

where $\chi_{ij} =< f_{i\sigma}^\dagger f_{j\sigma} + \frac{2t}{J_p}(b_{i1}^\dagger b_{j1} - b_{j2}^\dagger b_{i2}) >$, $\Delta_{ij}^f =< f_{j1}f_{i2} - f_{j2}f_{i1} >$ and $\mu_i^b = -\mu - \frac{J}{2}\sum_{j=i\pm\hat{x},i\pm\hat{y}} |\Delta_{ij}^f|^2$. Taking the saddle point value for the Lagrangian multiplier fields $\lambda_i^l = 0$ [15], we obtain the holon Hamiltonian, Eq.(3).

APPENDIX C: DERIVATION OF EQS.(4) AND (5)

Introducing a uniform hopping order parameter, $\chi_{ji} = \chi$, and a d-wave spinon pairing order parameters, $\Delta_{ji}^f = \pm\Delta_f$(the sign $+(-)$ is for the $\mathbf{ij}$ link parallel to $\hat{x}$ $(\hat{y})$), we obtain the energy-momentum space representation of the action for the U(1) theory,

$$S_{U(1)}^b = \sum_{\mathbf{k},\omega_n} (-i\omega_n + \epsilon(\mathbf{k}))b(\mathbf{k},\omega_n)^\dagger b(\mathbf{k},\omega_n)$$
$$+\frac{1}{2N\beta}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}\sum_{\omega_n,\omega_n',\nu_n} v(\mathbf{k}-\mathbf{k}')b(-\mathbf{k}'+\mathbf{q},-\omega_n'+\nu_n)^\dagger b(\mathbf{k}',\omega_n')^\dagger b(\mathbf{k},\omega_n)b(-\mathbf{k}+\mathbf{q},-\omega_n+\nu_n), \quad \text{(C1)}$$

where $b(\mathbf{k},\omega_n) = \frac{1}{\sqrt{N\beta}}\int_0^\beta d\tau \sum_i e^{i(\omega_n\tau - \mathbf{k}\cdot\mathbf{r}_i)} b_i(\tau)$, $\epsilon(\mathbf{k}) = -2t\chi\gamma_{\mathbf{k}} - \mu$, the energy dispersion of holon with $\gamma_{\mathbf{k}} = (\cos k_x + \cos k_y)$ and $v(\mathbf{k}' - \mathbf{k}) = -J|\Delta_f|^2\gamma_{\mathbf{k}}$, the momentum space representation of holon-holon interaction. N is the total number of sites for the two dimensional lattice of interest and $\beta = \frac{1}{k_BT}$, the inverse temperature. Considering the ladder diagrams for the holon-holon scattering, we obtain the Bethe-Salpeter equation for the t-matrix,

$$\begin{aligned}
&< k', -k'+q|t|k,-k+q >_{U(1)} = v(\mathbf{k}'-\mathbf{k}) \\
&-\frac{1}{N\beta}\sum_{k''} v(\mathbf{k}'-\mathbf{k}'')g(k'')g(-k''+q) \times \\
&< k'', -k''+q|t|k,-k+q >_{U(1)}, \qquad \text{(C2)}
\end{aligned}$$

where $g(k) =< b(\mathbf{k},\omega_n)b(\mathbf{k},\omega_n)^\dagger >$, the holon Matsubara Green's function and $k = (\omega_n, k_x, k_y)$ is the three-component vector of the energy and momentum. Using the fact that the holon-holon interaction $v(\mathbf{k}' - \mathbf{k}'')$ is frequency independent, we sum the Matsubara frequency k_0'',

$$\begin{aligned}
\frac{1}{\beta}\sum_{k_0''} g(k'')g(-k''+q) &= \frac{1}{\beta}\sum_{k_0''}\frac{1}{ik_0''-\epsilon(\mathbf{k}'')}\frac{1}{i(-k_0''+q_0)-\epsilon(-\mathbf{k}''+\mathbf{q})} \\
&= -\frac{n(\epsilon(\mathbf{k}''))+e^{\beta\epsilon(-\mathbf{k}''+\mathbf{q})}n(\epsilon(-\mathbf{k}''+\mathbf{q}))}{iq_0-(\epsilon(-\mathbf{k}''+\mathbf{q})+\epsilon(\mathbf{k}''))}. \qquad \text{(C3)}
\end{aligned}$$

Inserting Eq.(C3) into Eq.(C2) and defining $t_{\mathbf{k}',\mathbf{k}}(\mathbf{q},q_0) \equiv < k', -k'+q|t|k,-k+q >_{U(1)}$, we obtain the Bethe-Salpeter equation for the t-matrix,

$$\sum_{\mathbf{k}''} \left(\delta_{\mathbf{k}',\mathbf{k}''} - m_{\mathbf{k}',\mathbf{k}''}(\mathbf{q},q_0)\right) t_{\mathbf{k}'',\mathbf{k}}(\mathbf{q},q_0) = v(\mathbf{k}'-\mathbf{k}), \tag{C4}$$

where

$$\begin{aligned} m_{\mathbf{k}',\mathbf{k}''}(\mathbf{q},q_0) &\equiv -\frac{1}{N\beta}\sum_{k_0''} v(\mathbf{k}'-\mathbf{k}'')g(k'')g(-k''+q) \\ &= -\frac{v(\mathbf{k}'-\mathbf{k}'')}{N\beta}\left[\sum_{k_0''}\frac{1}{ik_0''-\epsilon(\mathbf{k}'')}\frac{1}{i(-k_0''+q_0)-\epsilon(-\mathbf{k}''+\mathbf{q})}\right] \\ &= \frac{1}{N}v(\mathbf{k}'-\mathbf{k}'')\frac{n(\epsilon(\mathbf{k}''))+e^{\beta\epsilon(-\mathbf{k}''+\mathbf{q})}n(\epsilon(-\mathbf{k}''+\mathbf{q}))}{iq_0-(\epsilon(-\mathbf{k}''+\mathbf{q})+\epsilon(\mathbf{k}''))} \end{aligned} \tag{C5}$$

with $n(\epsilon) = \frac{1}{e^{\beta\epsilon}-1}$, the boson distribution function.

APPENDIX D: DERIVATION OF EQS.(6) AND (7)

Introducing a uniform hopping order parameter, $\chi_{ji} = \chi$ and a d-wave spinon pairing order parameters, $\Delta^f_{ji} = \pm\Delta_f$(the sign $+(-)$ is for the $\mathbf{ij}$ link parallel to $\hat{x}$ $(\hat{y})$), we obtain the energy-momentum space representation of the action for the SU(2) theory,

$$\begin{aligned} S^b_{SU(2)} &= \sum_{\mathbf{k},\omega_n}\left(b_1(\mathbf{k},\omega_n)^\dagger, b_2(\mathbf{k},\omega_n)^\dagger\right)\begin{pmatrix} -i\omega_n - t\chi\gamma_{\mathbf{k}} - \mu & t\Delta_f\varphi_{\mathbf{k}} \\ t\Delta_f\varphi_{\mathbf{k}} & -i\omega_n + t\chi\gamma_{\mathbf{k}} - \mu \end{pmatrix}\begin{pmatrix} b_1(\mathbf{k},\omega_n) \\ b_2(\mathbf{k},\omega_n)\end{pmatrix} \\ &+\frac{1}{2N}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\alpha,\beta}\sum_{\omega_n,\omega_n',\nu_n} v(\mathbf{k}-\mathbf{k}')b_\beta(-\mathbf{k}'+\mathbf{q},-\omega_n'+\nu_n)^\dagger b_\alpha(\mathbf{k}',\omega_n')^\dagger b_\alpha(\mathbf{k},\omega_n)b_\beta(-\mathbf{k}+\mathbf{q},-\omega_n+\nu_n), \end{aligned} \tag{D1}$$

where $b_\alpha(\mathbf{k},\omega_n) = \frac{1}{\sqrt{N\beta}}\int_0^\beta d\tau \sum_i e^{i(\omega_n\tau - \mathbf{k}\cdot\mathbf{r}_i)}b_{i\alpha}(\tau)$ and $\varphi_{\mathbf{k}} = (\cos k_x - \cos k_y)$ with α, β $(= 1,2)$, two components of SU(2) holon. Considering the ladder diagrams for the holon-holon scattering, we obtain the Bethe-Salpeter equation for the t-matrix,

$$\begin{aligned} &< k',\alpha'; -k'+q,\beta'|t|k,\alpha;-k+q,\beta >_{SU(2)} = v(\mathbf{k}'-\mathbf{k})\delta_{\alpha',\alpha}\delta_{\beta',\beta} \\ &-\frac{1}{N\beta}\sum_{k'',\alpha'',\beta''} v(\mathbf{k}'-\mathbf{k}'')g_{\alpha'\alpha''}(k'')g_{\beta'\beta''}(-k''+q) \\ &\times < k'',\alpha''; -k''+q,\beta''|t|k,\alpha;-k+q,\beta >_{SU(2)}, \end{aligned} \tag{D2}$$

where $g_{\alpha\beta}(k) =< b_\alpha(\mathbf{k},\omega_n) b_\beta(\mathbf{k},\omega_n)^\dagger >$, the holon Matsubara Green's function.

To calculate the holon Matsubara Green's function in the diagonalized basis of the one-body term in the action (Eq.(D1)), we introduce the unitary transformation for the holon field,

$$\begin{pmatrix} b_1(\mathbf{k},\omega_n) \\ b_2(\mathbf{k},\omega_n) \end{pmatrix} = U(\mathbf{k}) \begin{pmatrix} b_1^{'}(\mathbf{k},\omega_n) \\ b_2^{'}(\mathbf{k},\omega_n) \end{pmatrix} \tag{D3}$$

where

$$U_{\alpha\beta}(\mathbf{k}) = \begin{pmatrix} u_\mathbf{k} & -v_\mathbf{k} \\ v_\mathbf{k} & u_\mathbf{k} \end{pmatrix} \tag{D4}$$

with $u_\mathbf{k} = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{t\chi\gamma_\mathbf{k}}{E_\mathbf{k}}}$, $v_\mathbf{k} = \frac{sgn(\varphi_k)}{\sqrt{2}}\sqrt{1 + \frac{t\chi\gamma_\mathbf{k}}{E_\mathbf{k}}}$, $E(\mathbf{k}) = t\sqrt{(\chi\gamma_\mathbf{k})^2 + (\Delta_f\varphi_\mathbf{k})^2}$ and $b_\alpha^{'}(\mathbf{k},\omega_n)$, the quasi-holon field. Then the one-body action in Eq.(D1) becomes

$$S^{b0}_{SU(2)} = \sum_{\mathbf{k},\omega_n} \left(b_1(\mathbf{k},\omega_n)^\dagger, b_2(\mathbf{k},\omega_n)^\dagger\right) \begin{pmatrix} -i\omega_n - t\chi\gamma_\mathbf{k} - \mu & t\Delta_f\varphi_\mathbf{k} \\ t\Delta_f\varphi_\mathbf{k} & -i\omega_n + t\chi\gamma_\mathbf{k} - \mu \end{pmatrix} \begin{pmatrix} b_1(\mathbf{k},\omega_n) \\ b_2(\mathbf{k},\omega_n) \end{pmatrix}$$
$$= \sum_{\mathbf{k},\omega_n} \left(b_1^{'}(\mathbf{k},\omega_n)^\dagger, b_2^{'}(\mathbf{k},\omega_n)^\dagger\right) \begin{pmatrix} -i\omega_n + E_1(\mathbf{k}) & 0 \\ 0 & -i\omega_n + E_2(\mathbf{k}) \end{pmatrix} \begin{pmatrix} b_1^{'}(\mathbf{k},\omega_n) \\ b_2^{'}(\mathbf{k},\omega_n) \end{pmatrix}, \tag{D5}$$

where the quasi-holon energy is $E_1(\mathbf{k}) = t\sqrt{(\chi\gamma_\mathbf{k})^2 + (\Delta_f\varphi_\mathbf{k})^2} - \mu$ and $E_2(\mathbf{k}) = -t\sqrt{(\chi\gamma_\mathbf{k})^2 + (\Delta_f\varphi_\mathbf{k})^2} - \mu$. From the action in Eq.(D5) we readily obtain the holon Matsubara Green's function,

$$\begin{aligned} g_{\alpha\beta}(k) &=< b_\alpha(\mathbf{k},\omega_n) b^\dagger_\beta(\mathbf{k},\omega_n) > \\ &= U_{\alpha\alpha'}(\mathbf{k}) < b^{'}_{\alpha'}(\mathbf{k},\omega_n) b^{'\dagger}_{\beta'}(\mathbf{k},\omega_n) > (U(\mathbf{k})^\dagger)_{\beta'\beta} \\ &= U_{\alpha\alpha'}(\mathbf{k}) \begin{pmatrix} -\frac{1}{i\omega_n + E_1(\mathbf{k})} & 0 \\ 0 & -\frac{1}{i\omega_n + E_2(\mathbf{k})} \end{pmatrix}_{\alpha'\beta'} (U(\mathbf{k})^\dagger)_{\beta'\beta}, \end{aligned} \tag{D6}$$

where $E_1(\mathbf{k})$ and $E_2(\mathbf{k})$ are the holon quasiparticle energy in the upper and lower band respectively and $\mathrm{sgn}(\varphi_k)$ denotes the sign of $(\cos k_x - \cos k_y)$. It is of note that there are two energy bands owing to the two kinds of holons in the SU(2) theory.

Using the fact that the holon-holon interaction $v(\mathbf{k}^{'} - \mathbf{k}^{''})$ is frequency independent, we sum the Matsubara frequency $k_0^{''}$ in Eq.(D2),

$$
\begin{aligned}
&\frac{1}{\beta}\sum_{k_0^{''}} g_{\alpha'\alpha''}(k^{''})g_{\beta'\beta''}(-k^{''}+q) \\
&= \frac{1}{\beta}\sum_{k_0^{''}}\sum_{\alpha_1,\beta_1}\left(U(\mathbf{k})_{\alpha'\alpha_1}\frac{1}{ik_0^{''}-E_{\alpha_1}(\mathbf{k}^{''})}U(\mathbf{k})^{\dagger}_{\alpha_1\alpha''}\right) \\
&\times\left(U(-\mathbf{k}+\mathbf{q})_{\beta'\beta_1}\frac{1}{i(-k_0^{''}+q_0)-E_{\beta_1}(-\mathbf{k}''+\mathbf{q})}U^{\dagger}(-\mathbf{k}+\mathbf{q})_{\beta_1\beta''}\right) \\
= &-U(\mathbf{k})_{\alpha'\alpha_1}U(\mathbf{k})^{\dagger}_{\alpha_1\alpha''}U(-\mathbf{k}+\mathbf{q})_{\beta'\beta_1}U^{\dagger}(-\mathbf{k}+\mathbf{q})_{\beta_1\beta''} \\
&\times\frac{n(E_{\alpha_1}(\mathbf{k}^{''}))+e^{\beta E_{\beta_1}(-\mathbf{k}^{''}+\mathbf{q})}n(E_{\beta_1}(-\mathbf{k}^{''}+\mathbf{q}))}{iq_0-(E_{\beta_1}(-\mathbf{k}''+\mathbf{q})+E_{\alpha_1}(\mathbf{k}''))}.
\end{aligned} \tag{D7}
$$

Inserting Eq.(D7) into Eq.(D2) and defining $t^{\alpha^{'}\beta^{'}\alpha\beta}_{\mathbf{k}^{'},\mathbf{k}}(\mathbf{q},q_0)\equiv<k^{'},\alpha^{'};-k^{'}+q,\beta^{'}|t|k,\alpha;-k+q,\beta>_{SU(2)}$, we obtain the Bethe-Salpeter equation for the t-matrix,

$$
\begin{aligned}
&\sum_{\mathbf{k}'',\alpha'',\beta''}\left(\delta_{\mathbf{k}',\mathbf{k}''}\delta_{\alpha'\alpha''}\delta_{\beta'\beta''}-m^{\alpha^{'}\beta^{'}\alpha^{''}\beta^{''}}_{\mathbf{k}',\mathbf{k}''}(\mathbf{q},q_0)\right)t^{\alpha^{''}\beta^{''}\alpha\beta}_{\mathbf{k}'',\mathbf{k}}(\mathbf{q},q_0) \\
&= v(\mathbf{k}^{'}-\mathbf{k})\delta_{\alpha'\alpha}\delta_{\beta'\beta},
\end{aligned} \tag{D8}
$$

where

$$
\begin{aligned}
&m^{\alpha^{'}\beta^{'}\alpha^{''}\beta^{''}}_{\mathbf{k}',\mathbf{k}''}(\mathbf{q},q_0)=\frac{1}{N}\sum_{\alpha_1^{'}\beta_1^{'}}v(\mathbf{k}^{'}-\mathbf{k}^{''})\frac{n(E_{\alpha_1^{'}}(\mathbf{k}^{''}))+e^{\beta E_{\beta_1^{'}}(-\mathbf{k}^{''}+\mathbf{q})}n(E_{\beta_1^{'}}(-\mathbf{k}^{''}+\mathbf{q}))}{iq_0-(E_{\alpha_1^{'}}(\mathbf{k}'')+E_{\beta_1^{'}}(-\mathbf{k}''+\mathbf{q}))}\times \\
&U_{\alpha'\alpha_1^{'}}(\mathbf{k}^{''})U_{\beta'\beta_1^{'}}(-\mathbf{k}^{''}+\mathbf{q})U^{\dagger}_{\alpha_1^{'}\alpha''}(\mathbf{k}^{''})U^{\dagger}_{\beta_1^{'}\beta''}(-\mathbf{k}^{''}+\mathbf{q}).
\end{aligned} \tag{D9}
$$

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[22] Using the U(1) slave-boson representation of the electron annihilation operator $c_{i\sigma} = f_{i\sigma} b_i^\dagger$, the Cooper pair order parameter is decomposed into the holon and spinon pairing order parameters, $< \sigma c_{i\sigma} c_{j-\sigma} > = < b_i^\dagger b_j^\dagger > < \sigma f_{i\sigma} f_{j-\sigma} >$. Thus, the s-wave symmetry of holon pairing ($< b_i^\dagger b_{i+\hat{x}}^\dagger > = < b_i^\dagger b_{i+\hat{y}}^\dagger >$) and the d-wave symmetry of spinon pairing ($< \sigma f_{i\sigma} f_{i+\hat{x}-\sigma} > = - < \sigma f_{i\sigma} f_{i+\hat{y}-\sigma} >$) leads to the d-wave symmetry of Cooper pairing ($< \sigma c_{i\sigma} c_{i+\hat{x}-\sigma} > = - < \sigma c_{i\sigma} c_{i+\hat{y}-\sigma} >$).

[23] In the mean-field approximation, the Cooper pair order parameter is decomposed into the holon and spinon contributions,

$$\begin{aligned} < \sigma c_{i\sigma} c_{j-\sigma} > \; &= \frac{1}{2}\Big[< b_{i1}^\dagger b_{j1}^\dagger > < \sigma f_{i\sigma} f_{j-\sigma} > - < b_{i2}^\dagger b_{j2}^\dagger > < \sigma f_{i\sigma} f_{j-\sigma} >^* \\ &\quad + < b_{i2}^\dagger b_{j1}^\dagger > < f_{i\sigma}^\dagger f_{j\sigma} > + < b_{i1}^\dagger b_{j2}^\dagger > < f_{i\sigma}^\dagger f_{j\sigma} >^* \Big] \end{aligned} \tag{D10}$$

with i and j are the nearest neighbor sites. Here, we used the SU(2) representation [21] of electron annihilation operator, that is, $c_{i\uparrow} = \frac{1}{\sqrt{2}}(b_{i1}^\dagger f_{i1} + b_{i2}^\dagger f_{i2}^\dagger)$ for spin up electron and $c_{i\downarrow} = \frac{1}{\sqrt{2}}(b_{i1}^\dagger f_{i2} - b_{i2}^\dagger f_{i1}^\dagger)$ for spin down electron. In the real space, the first eigenvector in Table 2 is written $< b_{i\alpha}^\dagger b_{j\beta}^\dagger > = (\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$ for $j = i \pm \hat{x}$ or $j = i \pm \hat{y}$. Combining with the d-wave symmetry of the spinon pairing order parameter, the first two terms in Eq.(D10) cancel. This results in the vanishing Cooper pair order parameter. On the other hand, the second eigenvector is written, in real space, $< b_{i\alpha}^\dagger b_{j\beta}^\dagger > = (\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) \mp a(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$, where $-(+)$ sign is for $j = i \pm \hat{x}$ ($j = i \pm \hat{y}$). This results

in a non-vanishing Cooper pair order parameter with pure $d_{x^2-y^2}$ symmetry. The first two terms in Eq.(D10) yield the d-wave symmetry of Cooper pairs as a composite of the s-wave symmetry of holon pair ($< b_{i\alpha} b_{j\alpha} >$) and the d-wave symmetry of spinon pair ($< \sigma f_{i\sigma} f_{j-\sigma} >$). The last two terms in Eq.(D10) also lead to the d-wave symmetry of Cooper pair owing to the d-wave symmetry of the b_1-b_2 holon pair ($< b_{i1} b_{j2} >$) and the uniform phase of the spinon hopping order parameter ($< f^\dagger_{i\sigma} f_{j\sigma} >$).

[24] For instance, the lowest energy pole ($0.0111t$) for $x = 0.04$ in the U(1) theory approximately equals the kinetic energy of holon pair with momenta $\mathbf{k} = (0,0)$, i.e., $2\epsilon(0,0) = 0.016t$ with t, the hopping energy as is shown in Table. 4. The energies of the next four higher lying states are predicted to be reasonably close to the kinetic energies of holon pair corresponding to momenta $\mathbf{k} = (\pm 2\pi/L, 0)$ or $(0, \pm 2\pi/L)$ (i.e., $\epsilon(2\pi/L, 0) + \epsilon(-2\pi/L, 0) = \epsilon(0, 2\pi/L) + \epsilon(0, -2\pi/L) = 0.724t$) as shown in Table 4. (Here $2\pi/L$ is the smallest possible nonzero momentum of k_x or k_y for $L \times L$ square lattice.) Similarly the energies of the next three higher lying states are also close to the kinetic energy, $\epsilon(2\pi/L, 2\pi/L) + \epsilon(-2\pi/L, -2\pi/L) = 1.432t$; the next four higher lying states have energies which are approximately the same as the kinetic energy, $\epsilon(4\pi/L, 0) + \epsilon(-4\pi/L, 0) = \epsilon(0, 4\pi/L) + \epsilon(0, -4\pi/L) = 2.578t$. Similar arguments hold true for other hole concentrations in Table 4. The SU(2) results are shown in Table 5, revealing a trend similar to the U(1) case above.

[25] For the p_x-wave pairing state, $w_l(\mathbf{k}) = \sin k_x$, the transition amplitude is similarly obtained to be

$$T_{p_x \to \{\mathbf{k}_1, -\mathbf{k}_1\}} = -\frac{N}{2} J |\Delta_f|^2 \sin k_{1x}. \tag{D11}$$

For $\sin k_{1x} = 0$, the transition does not occur from the pairing state of p_x orbital to any intermediate state with momenta $\mathbf{k}_1$ and $-\mathbf{k}_1$. This explains why there is no eigenvector of p_x or p_y at the lowest energy pole which comes from the holon of momentum $(0,0)$. Similarly, for the d-wave pairing state, $w_l(\mathbf{k}) = (\cos k_x - \cos k_y)$, the lowest order transition amplitude is given by,

$$T_{d \to \{\mathbf{k}_1, -\mathbf{k}_1\}} = -\frac{N}{2} J |\Delta_f|^2 (\cos k_{1x} - \cos k_{1y}). \tag{D12}$$

The transition amplitude vanishes if $k_x = \pm k_y$. This implies that there is no d-wave eigenvector at the pole near the energy of $2\epsilon(0,0)$, $2\epsilon(2\pi/L, 2\pi/L)$. This is because the d-wave pairing state can not make transitions into intermediate states having momenta $\mathbf{k}_1$ and $-\mathbf{k}_1$ where $\mathbf{k}_1 = (0,0)$ or $(2\pi/L, 2\pi/L)$.

TABLE CAPTIONS

Table 1 Weights of the s-, p_x-, p_y- and d-wave contributions to the holon pairing order parameter corresponding to the lowest energy in the U(1) holon-pair boson theory.

Table 2 Weights of the s-, p_x-, p_y- and d-wave contributions in each b_α-b_β scattering channel with $\alpha, \beta = 1, 2$ to the first holon pair order parameter corresponding to the lowest energy in the SU(2) holon-pair boson theory.

Table 3 Weights of the s-, p_x-, p_y- and d-wave contributions in each b_α-b_β scattering channel with $\alpha, \beta = 1, 2$ to the second holon pair order parameter corrsponding to the lowest energy in the SU(2) holon-pair boson theory.

Table 4 The orbital state and energy at various hole doping x and temperatures ($T/t = T_c/t + 0.001$) for underdoping ($x = 0.04$, $T_c/t = 0.034$), optimal doping($x = 0.07$, $T_c/t = 0.044$) and overdoping ($x = 0.1$, $T_c/t = 0.041$) respectively with the U(1) holon-pair boson theory. The calculations are done on the $N = 10 \times 10$ lattice with the use of $J/t = 0.3$. Here, $\mathbf{s}$ denotes $(\cos k_x + \cos k_y)$; $\mathbf{d}$, $(\cos k_x - \cos k_y)$; $\mathbf{p_x}$, $\sin k_x$ and $\mathbf{p_y}$, $\sin k_y$.

Table 5 The orbital and energy at temperatures $T/t = T_c/t + 0.001$ for underdoping ($x = 0.07$, $T_c/t = 0.027$), optimal doping($x = 0.13$, $T_c/t = 0.037$) and overdoping ($x = 0.19$, $T_c/t = 0.031$) respectively in the SU(2) holon-pair boson theory.

TABLES

weight	$x = 0.04$	$x = 0.07$	$x = 0.1$
a_s	**1**	**1**	**1**
a_{p_x}	-1.8×10^{-14}	-2.5×10^{-14}	-5.3×10^{-15}
a_{p_y}	-1.4×10^{-14}	-1.8×10^{-14}	-3.4×10^{-14}
a_d	-1.3×10^{-15}	-8.9×10^{-15}	-3.2×10^{-15}

TABLE I.

weight	$x = 0.07$	$x = 0.13$	$x = 0.19$
$a_{s,11}$	**1**	**1**	**1**
$a_{p_x,11}$	-5.7×10^{-15}	-1.3×10^{-20}	6.1×10^{-21}
$a_{p_y,11}$	-2.4×10^{-14}	1.2×10^{-21}	-1.3×10^{-21}
$a_{d,11}$	-1.3×10^{-15}	-3.5×10^{-27}	-2.9×10^{-21}
$a_{s,12}$	9.2×10^{-20}	1.8×10^{-22}	4.9×10^{-18}
$a_{p_x,12}$	-6.7×10^{-27}	1.2×10^{-33}	6.2×10^{-25}
$a_{p_y,12}$	2.2×10^{-27}	6.6×10^{-28}	-5.9×10^{-31}
$a_{d,12}$	-4.3×10^{-26}	-2.2×10^{-33}	-3.5×10^{-31}
$a_{s,21}$	5.3×10^{-22}	1.8×10^{-22}	4.9×10^{-18}
$a_{p_x,21}$	-9.3×10^{-29}	1.2×10^{-33}	6.1×10^{-25}
$a_{p_y,21}$	3.1×10^{-29}	6.6×10^{-28}	-5.8×10^{-31}
$a_{d,21}$	-2.5×10^{-28}	1.4×10^{-29}	1.4×10^{-29}
$a_{s,22}$	**1**	**1**	**1**
$a_{p_x,22}$	1.0×10^{-20}	7.2×10^{-21}	-1.1×10^{-20}
$a_{p_y,22}$	1.2×10^{-20}	-5.2×10^{-22}	-6.2×10^{-21}
$a_{d,22}$	7.7×10^{-21}	-2.0×10^{-28}	1.6×10^{-27}

TABLE II.

weight	$x = 0.07$	$x = 0.13$	$x = 0.19$
$a_{s,11}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$a_{p_x,11}$	-4.0×10^{-15}	-7.2×10^{-21}	1.1×10^{-20}
$a_{p_y,11}$	-4.4×10^{-15}	5.2×10^{-22}	6.2×10^{-21}
$a_{d,11}$	-1.5×10^{-14}	1.9×10^{-28}	-1.6×10^{-27}
$a_{s,12}$	-4.9×10^{-14}	1.6×10^{-15}	6.8×10^{-22}
$a_{p_x,12}$	-2.7×10^{-14}	2.3×10^{-15}	3.2×10^{-27}
$a_{p_y,12}$	-4.9×10^{-14}	1.6×10^{-22}	-3.7×10^{-34}
$a_{d,12}$	$\mathbf{-1.3 \times 10^{-4}}$	$\mathbf{-8.3 \times 10^{-6}}$	-1.4×10^{-33}
$a_{s,21}$	-4.7×10^{-14}	7.5×10^{-11}	6.8×10^{-22}
$a_{p_x,21}$	2.6×10^{-14}	-1.9×10^{-10}	3.2×10^{-27}
$a_{p_y,21}$	1.7×10^{-14}	2.3×10^{-23}	-3.7×10^{-34}
$a_{d,21}$	$\mathbf{-1.3 \times 10^{-4}}$	$\mathbf{-8.3 \times 10^{-6}}$	-1.4×10^{-29}
$a_{s,22}$	$\mathbf{-1}$	$\mathbf{-1}$	$\mathbf{-1}$
$a_{p_x,22}$	7.1×10^{-15}	2.5×10^{-14}	6.3×10^{-21}
$a_{p_y,22}$	3.1×10^{-14}	1.2×10^{-20}	2.7×10^{-21}
$a_{d,22}$	2.5×10^{-15}	1.0×10^{-14}	-1.9×10^{-27}

TABLE III.

eigenvector	ω/t ($x = 0.04$)	ω/t ($x = 0.07$)	ω/t ($x = 0.1$)
$\mathbf{s}$	0.0111	0.0089	0.0063
$\mathbf{s}$	0.7221	1.1180	1.5592
$\mathbf{p_x}$, $\mathbf{p_y}$	0.7236	1.1186	1.5594
$\mathbf{d}$	0.7238	1.1187	1.5595
$\mathbf{s}$	1.4306	2.2249	3.1108
$\mathbf{p_x}$, $\mathbf{p_y}$	1.4316	2.2252	3.1109
$\mathbf{s}$	2.5769	4.0157	5.6212
$\mathbf{p_x}$, $\mathbf{p_y}$	2.5773	4.01585	5.62126
$\mathbf{d}$	2.5775	4.01594	5.62129

TABLE IV.

eigenvector	ω/t ($x = 0.07$)	ω/t ($x = 0.13$)	ω/t ($x = 0.19$)
$\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	0.0097 ($a = 1.3 \times 10^{-4}$)	0.0080 ($a = 8.3 \times 10^{-6}$)	0.0054 ($a < 10^{-10}$)
$\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	0.3494 ($a = 4 \times 10^{-3}$)	0.5899 ($a = 1.4 \times 10^{-3}$)	0.8935 ($a = 4.4 \times 10^{-4}$)
$\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) + a\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$, $\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	0.3510 ($a = 3.6 \times 10^{-2}$)	0.5905 ($a = 1.3 \times 10^{-2}$)	0.8936 ($a = 4.1 \times 10^{-3}$)
$\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	0.3512 ($a = 3.4 \times 10^{-1}$)	0.5906 ($a = 1.2 \times 10^{-1}$)	0.8997 ($a = 3.9 \times 10^{-2}$)
$\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	0.6910 ($a = 2.5 \times 10^{-4}$)	1.1707 ($a = 1.8 \times 10^{-5}$)	1.7809 ($a < 10^{-6}$)
$\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) + a\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$, $\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	0.6920 ($a = 2.6 \times 10^{-4}$)	1.1711 ($a = 2.0 \times 10^{-5}$)	1.7810 ($a < 10^{-6}$)
$\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	1.2030 ($a = 9.7 \times 10^{-2}$)	2.1015 ($a = 3.4 \times 10^{-2}$)	3.2154 ($a = 1.1 \times 10^{-2}$)
$\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) + a\mathbf{p_x}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$, $\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{p_y}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	1.1203 ($a = 1.7 \times 10^{-1}$)	2.1017 ($a = 6.4 \times 10^{-2}$)	3.2155 ($a = 2.1 \times 10^{-2}$)
$\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,1} + \delta_{\alpha,2}\delta_{\beta,2})$, $\mathbf{d}(\delta_{\alpha,1}\delta_{\beta,1} - \delta_{\alpha,2}\delta_{\beta,2}) - a\mathbf{s}(\delta_{\alpha,1}\delta_{\beta,2} + \delta_{\alpha,2}\delta_{\beta,1})$	1.1236 ($a = 3.4 \times 10^{-1}$)	2.1018 ($a = 1.2 \times 10^{-1}$)	3.2155 ($a = 3.9 \times 10^{-2}$)

TABLE V.

FIGURE CAPTIONS

Fig. 1 The schematic spectrum of the holon pairing state at temperature (a) above T_c and (b) below T_c.

Fig. 2 The lowest order t-matrix starting from an l-th orbital state $w_l(k)$ to an intermediate state of momenta $\mathbf{k}_1$ and $-\mathbf{k}_1$.

Fig. 3 The treated t-matrix with the inclusion of exchange contribution.

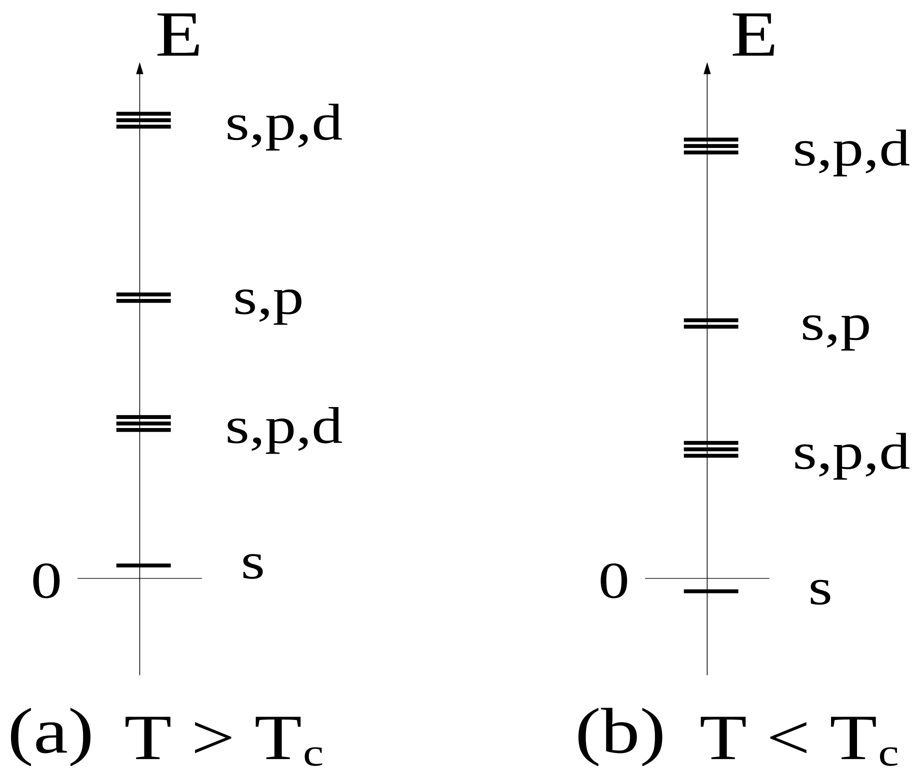

FIG. 1.

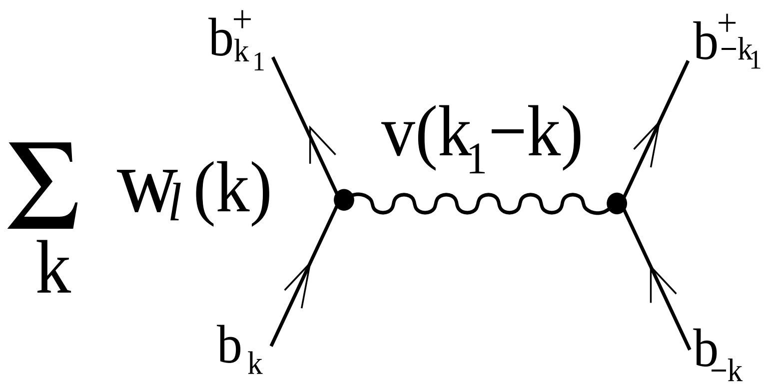

FIG. 2.

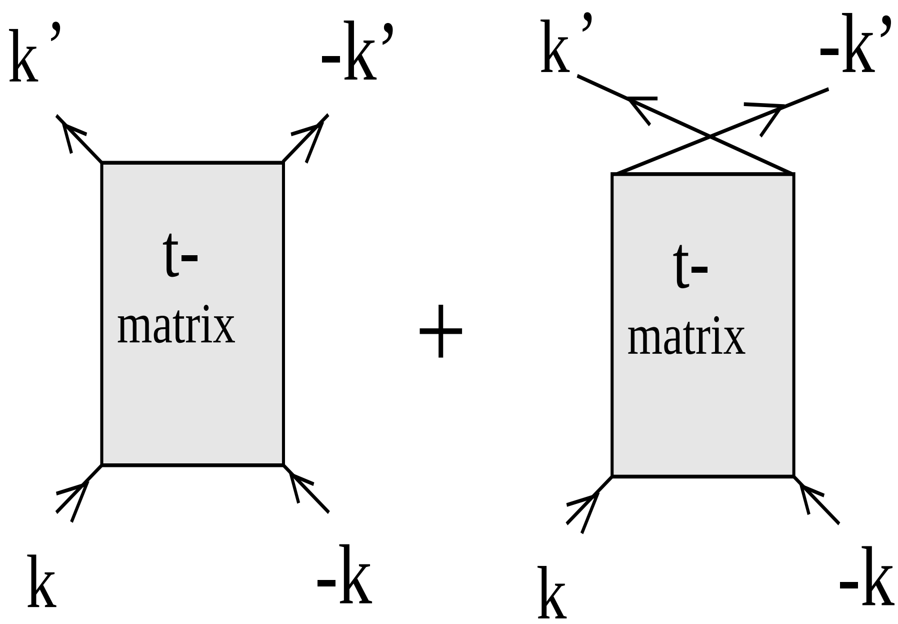

FIG. 3.